1. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(a) the coordinates of C

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(b) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(c) Hence, or otherwise, find the area of the triangle ABC.

(5)
(Total 12 marks)

2. The line l_1 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$$

and the line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} -6 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where λ and μ are parameters.

The lines l_1 and l_2 intersect at the point A and the acute angle between l_1 and l_2 is θ.

(a) Write down the coordinates of A.

(1)

(b) Find the value of $\cos \theta$.

(3)

The point X lies on l_1 where $\lambda = 4$.

(c) Find the coordinates of X.

(1)

(d) Find the vector $\overrightarrow{AX}$

(2)

(e) Hence, or otherwise, show that $\left|\overrightarrow{AX}\right| = 4\sqrt{26}.$

(2)

The point y lies on l_2. Given that the vector $\overrightarrow{YX}$ is perpendicular to l_1,

(f) find the length of AY, giving your answer to 3 significant figures.

(3)
(Total 12 marks)

3. Relative to a fixed origin O, the point A has position vector $(8\mathbf{i} + 13\mathbf{j} - 2\mathbf{k})$, the point B has position vector $(10\mathbf{i} + 14\mathbf{j} - 4\mathbf{k})$, and the point C has position vector $(9\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

The line l passes through the points A and B.

(a) Find a vector equation for the line l.

(3)

(b) Find $\left|\overrightarrow{CB}\right|$.

(2)

(c) Find the size of the acute angle between the line segment CB and the line l, giving your answer in degrees to 1 decimal place.

(3)

(d) Find the shortest distance from the point C to the line l.

(3)

The point X lies on l. Given that the vector $\overrightarrow{CX}$ is perpendicular to l,

(e) find the area of the triangle CXB, giving your answer to 3 significant figures.

(3)
(Total 14 marks)

4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)
(Total 13 marks)

5. The equations of the lines l_1 and l_2 are given by

$$l_1: \quad \mathbf{r} = \mathbf{i} + 3\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

$$l_2: \quad \mathbf{r} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}),$$

where λ and μ are parameters.

(a) Show that l_1 and l_2 intersect and find the coordinates of Q, their point of intersection.

(6)

(b) Show that l_1 is perpendicular to l_2.

(2)

The point P with x-coordinate 3 lies on the line l_1 and the point R with x-coordinate 4 lies on the line l_2.

(c) Find, in its simplest form, the exact area of the triangle PQR.

(6)
(Total 14 marks)

6. Relative to a fixed origin O, the point A has position vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, the point B has position vector $5\mathbf{i} + \mathbf{j} + \mathbf{k}$, and the point C has position vector $7\mathbf{i} - \mathbf{j}$.

(a) Find the cosine of angle ABC.

(4)

(b) Find the exact value of the area of triangle ABC.

(3)

The point D has position vector $7\mathbf{i} + 3\mathbf{k}$.

(c) Show that AC is perpendicular to CD.

(2)

(d) Find the ratio $AD : DB$.

(2)
(Total 11 marks)

7. The points A, B and C have position vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$ and $\mathbf{i} - \mathbf{j}$ respectively, relative to a fixed origin O.

(a) Prove that the points A, B and C lie on a straight line l.

(4)

The point D has position vector $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$.

(b) Find the cosine of the acute angle between l and the line OD.

(3)

The point E has position vector $-3\mathbf{j} - \mathbf{k}$.

(c) Prove that E lies on l and that OE is perpendicular to OD.

(4)
(Total 11 marks)

1. (a) **j** components $3+2\lambda=9 \quad \Rightarrow \lambda=3$ $\quad(\mu=1)$ M1 A1

Leading to $C:(5, 9, -1)$ accept vector forms A1 3

(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$ M1

$$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$$ use of scalar product M1 A1

$$\angle ACB = 57.95°$$ awrt $57.95°$ A1 4

Alternative method

A:(2, 3, –4) B:(–5, 9, –5) C:(5, 9, –1)

$AB^2 = 7^2 + 6^2 + 1^2 = 86$

$AC^2 = 3^2 + 6^2 + 3^2 = 54$

$BC^2 = 10^2 + 0^2 + 4^2 = 116$ Finding all three sides M1

$\cos\ \angle ACB = \dfrac{116+54-86}{2\sqrt{116}\sqrt{54}}$ (= 0.53066 ...) M1 A1

$\angle ACB$ = 57.95° awrt 57.95° A1 4

(c) $A:(2, 3, -4) \quad B:(-5, 9, -5)$

$$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix},\quad \overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$$

$AC^2 = 3^2+6^2+3^2 \quad\Rightarrow\quad AC = 3\sqrt{6}$ M1 A1

$BC^2 = 10^2+4^2 \quad\Rightarrow\quad BC = 2\sqrt{29}$ A1

$$\Delta ABC = \frac{1}{2}AC\times BC\sin\angle ACB$$

$$=\frac{1}{2}3\sqrt{6}\times 2\sqrt{29}\sin\angle ACB \approx 33.5$$ $15\sqrt{5}$, awrt 34 M1 A1 5

Alternative method

A:(2, 3, –4) B:(–5, 9, –5) C:(5, 9, –1)

$AB^2 = 7^2 + 6^2 + 1^2 = 86$

$AC^2 = 3^2 + 6^2 + 3^2 = 54$

$BC^2 = 10^2 + 0^2 + 4^2 = 116$ Finding all three sides M1

$\cos\ \angle ACB = \dfrac{116+54-86}{2\sqrt{116}\sqrt{54}}$ (= 0.53066 ...)		M1 A1	
$\angle ACB$ = 57.95°	awrt 57.95°	A1	4

If this method is used some of the working may gain credit in part (c) and appropriate marks may be awarded if there is an attempt at part (c).

[12]

2. (a) A: (–6, 4,–1) — Accept vector forms — B1 — 1

(b) $\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}3\\-4\\1\end{pmatrix}$ =12 + 4 + 3 =

$\sqrt{4^2+(-1)^2+3^2}\sqrt{3^2+(-4)^2+1^2}\cos\theta$ — M1 A1

$\cos\theta = \dfrac{19}{26}$ — awrt 0.73 — A1 — 3

(c) X: (10, 0,11) — Accept vector forms — B1 — 1

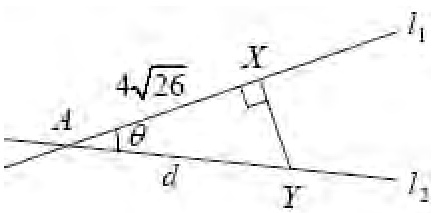
(d) $\overrightarrow{AX} = \begin{pmatrix}10\\0\\11\end{pmatrix} - \begin{pmatrix}-6\\4\\-1\end{pmatrix}$ — Either order — M1

$\begin{pmatrix}16\\-4\\12\end{pmatrix}$ — cao — A1 — 2

(e) $\left|\overrightarrow{AX}\right| = \sqrt{16^2+(-4)^2+12^2}$ — M1

$= \sqrt{416} = \sqrt{16\times 26} = 4\sqrt{26}$ * — Do not penalise if consistent incorrect signs in (d) — A1 — 2

(f)



Use of correct right angled triangle — M1

$\dfrac{|\overrightarrow{AX}|}{d} = \cos\theta$ M1

$d = \dfrac{4\sqrt{26}}{\frac{19}{26}} \approx 27.9$ awrt 27.9 A1 3

[12]

3. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}10\\14\\-4\end{pmatrix} - \begin{pmatrix}8\\13\\-2\end{pmatrix} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix}-2\\-1\\2\end{pmatrix}$ M1

$\mathbf{r} = \begin{pmatrix}8\\13\\-2\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ *or* $\mathbf{r} = \begin{pmatrix}10\\14\\-4\end{pmatrix} + \lambda\begin{pmatrix}2\\1\\-2\end{pmatrix}$ accept equivalents M1 A1ft 3

(b) $\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} = \begin{pmatrix}10\\14\\-4\end{pmatrix} - \begin{pmatrix}9\\9\\6\end{pmatrix} = \begin{pmatrix}1\\5\\-10\end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix}-1\\-5\\10\end{pmatrix}$

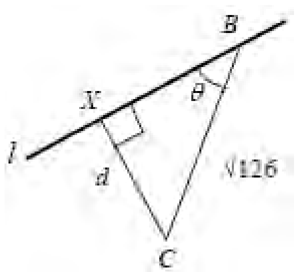
$CB = \sqrt{(1^2 + 5^2 + (-10)^2)} = \sqrt{(126)}\ (= 3\sqrt{14} \approx 11.2)$ awrt 11.2 M1 A1 2

(c) $\overrightarrow{CB}.\overrightarrow{AB} = |\overrightarrow{CB}||\overrightarrow{AB}|\cos\theta$

$(\pm)(2 + 5 + 20) = \sqrt{126}\sqrt{9}\cos\theta$ M1 A1

$\cos\theta = \dfrac{3}{\sqrt{14}} \Rightarrow \theta \approx 36.7°$ awrt 36.7° A1 3

(d)



$\dfrac{d}{\sqrt{126}} = \sin\theta$ M1 A1ft

$d = 3\sqrt{5}\ (\approx 6.7)$ awrt 6.7 A1 3

(e) $BX^2 = BC^2 - d^2 = 126 - 45 = 81$ M1

! $CBX = \frac{1}{2} \times BX \times d = \frac{1}{2} \times 9 \times 3\sqrt{5} = \frac{27\sqrt{5}}{2} (\approx 30.2)$ awrt 30.1 or 30.2 M1 A1 3

Alternative for (e)

! $CBX = \frac{1}{2} \times d \times BC \sin \angle XCB$ M1

$= \frac{1}{2} \times 3\sqrt{5} \times \sqrt{126} \sin (90 - 36.7)^{\circ}$ sine of correct angle M1

≈ 30.2 $\frac{27\sqrt{5}}{2}$, awrt 30.1 or 30.2 A1 3

[14]

4. (a) $\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\}$ = $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$ Apply

dot product calculation between two direction vectors, ie. $\underline{(-2 \times q) + (1 \times 2) + (-4 \times 2)}$ M1

$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$

$-2q = 6 \Rightarrow \underline{q = -3}$ AGand solves to find $q = -3$ A1 cso 2

(b) Lines meet where:

$$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

First two of
i: $11 - 2\lambda = -5 + q\mu$ (1)
j: $2 + \lambda = 11 + 2\mu$ (2)
k: $17 - 4\lambda = p + 2\mu$ (3)

Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.) M1

(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$ Attempts to solve (1) and (2) to find one of either λ or μ dM1

Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$ A1

Both $\lambda = 5$ or $\underline{\mu = -2}$ A1

(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$

(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$ Attempt to substitute their λ and μ into their **k** component to give an equation in p alone. ddM1

$\Rightarrow p = 17 - 20 + 4 \Rightarrow p = 1$ $\underline{p = -1}$ A1 cso 6

(c) $\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$ Substitutes their value of λ or μ into the correct line l_1 or l_2. M1

Intersect at $\mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or (1, 7, –3) $\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}$ or (1, 7, –3) A1 2

(d) Let $\overrightarrow{OX} = \mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ be point of intersection

$\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = \begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} - \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} = \begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ Finding vector $\overrightarrow{AX}$ by finding the difference between $\overrightarrow{OX}$ and $\overrightarrow{OA}$. M1ft ±

Can be ft using candidate's $\overrightarrow{OX}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + 2\overrightarrow{AX}$

$\overrightarrow{OB} = \begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\begin{pmatrix} -8 \\ 4 \\ -16 \end{pmatrix}$ $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix} + 2\left(\text{their } \overrightarrow{AX}\right)$ dM1ft

Hence, $\overrightarrow{OB} = \begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $\overrightarrow{OB} = -7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}$ $\begin{pmatrix} -7 \\ 11 \\ -19 \end{pmatrix}$ or $-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}$ or (–7, 11,–19) A1 3

[13]

5. (a) Any two of $1 + \lambda = -2 + 2\mu$ M1

$3 + 2\lambda = 3 + \mu$

$5 - \lambda = -4 + 4\mu$

Solve simultaneous equations to obtain $\mu = 2$, **or** $\lambda = 1$ M1 A1

$\therefore$ intersect at (2, 5, 4) M1 A1

Check in the third equation or on second line B1 6

(b) $1 \times 2 + 2 \times 1 + (-1) \times 4 = 0$ $\therefore$ perpendicular M1 A1 2

	(c)	P is the point (3, 7, 3) [i.e. $\lambda = 2$] and R is the point (4, 6, 8) [i.e. $\mu = 3$]	M1 A1	
		$PQ = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$		
		$RQ = \sqrt{2^2 + 1^2 + 4^2} = \sqrt{21}$	M1 A1	
		$PR = \sqrt{27}$		
		Need two of these for M1		
		The area of the triangle = $\frac{1}{2} \times \sqrt{6} \times \sqrt{21} = \frac{3\sqrt{14}}{2}$	M1 A1	6
		Or area = $\frac{1}{2} \times \sqrt{6} \times \sqrt{27} \sin P$ where $\sin P = \frac{\sqrt{7}}{3} = \frac{3\sqrt{14}}{2}$		
		Or area = $\frac{1}{2} \times \sqrt{21} \times \sqrt{27} \sin R$ where $\sin R = \frac{\sqrt{2}}{3} = \frac{3\sqrt{14}}{2}$ *(must be simplified)*		

[14]

6.	(a)	$\overrightarrow{AB} = (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, $\overrightarrow{CB} = (-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, (or $\overrightarrow{BA}$, $\overrightarrow{BC}$, or $\overrightarrow{AB}$, $\overrightarrow{BC}$ stated in above form or column vector form.	M1 A1	
		$\cos A\hat{B}C = \frac{CB \bullet AB}{\lvert CB \rvert \lvert AB \rvert} = -\frac{4}{9}$ or (–0.444)	M1 A1	4
	(b)	Area of $\Delta ABC = \frac{1}{2} \times 3 \times 2 \times \sin B$	M1	
		$\sin B = \sqrt{(1 - \frac{16}{81})} = \frac{\sqrt{65}}{9}$	M1	
		$\therefore$ Area = $\frac{1}{2}\sqrt{65}$	A1	3
	(c)	$\overrightarrow{AC} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ or given in alternative form with attempt at scalar product	M1	
		$\overrightarrow{AC} \bullet \overrightarrow{DC} = 0$, therefore the lines are perpendicular.	A1	2

	(d)	$\overrightarrow{AD} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$ and AD:DB = 2:–1 (allow 2:1)	M1, A1	2

[11]

7.	(a)	$\overrightarrow{AB}$ = 3**i** + 6**j** + 3**k**	B1	
		$\overrightarrow{AC}$ = (–**i** –2**j** –**k**)	B1	
		$= -\frac{1}{3}(3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) = -\frac{1}{3}\overrightarrow{AB}$	M1	
		Hence *A*, *B* and *C* are collinear	A1	4
	(b)	$\cos\theta = \frac{(2\mathbf{i}+\mathbf{j}-3\mathbf{k}).(\mathbf{i}+2\mathbf{j}+\mathbf{k})}{\sqrt{14}\times\sqrt{6}}$	M1 A1	
		$= \frac{1}{\sqrt{84}}$	A1	3
	(c)	$\overrightarrow{AE} = (-2\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) = -\frac{2}{3}\overrightarrow{AB}$	M1	
		so *E* is on *l*	A1	
		$\overrightarrow{OE}.\overrightarrow{OD} = (-3\mathbf{j} - \mathbf{k}).(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	M1	
		$= -3 + 3 = 0$		
		So $\overrightarrow{OE}$ and $\overrightarrow{OD}$ are perpendicular	A1	4

[11]

1. Part (a) was fully correct in the great majority of cases but the solutions were often unnecessarily long and nearly two pages of working were not unusual. The simplest method is to equate the j components. This gives one equation in λ, leading to $\lambda = 3$, which can be substituted into the equation of l_1 to give the coordinates of C. In practice, the majority of candidates found both λ and μ and many proved that the lines were coincident at C. However the question gave the information that the lines meet at C and candidates had not been asked to prove this. This appeared to be another case where candidates answered the question that they had expected to be set, rather than the one that actually had been.

The great majority of candidates demonstrated, in part (b), that they knew how to find the angle between two vectors using a scalar product. However the use of the position vectors of A and B, instead of vectors in the directions of the lines was common. Candidates could have used either the vectors $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}5\\0\\2\end{pmatrix}$, given in the question, or $\overrightarrow{AC}$ and $\overrightarrow{BC}$. The latter was much the commoner choice but many made errors in signs. Comparatively few chose to use the cosine rule. In part (c), many continued with the position vectors they had used incorrectly in part (b) and so found the area of the triangle OAB rather than triangle ABC. The easiest method of completing part (c) was usually to use the formula Area = $\frac{1}{2}ab\sin C$ and most chose this.

Attempts to use Area = $\frac{1}{2}$base × height were usually fallacious and often assumed that the triangle was isosceles. A few complicated attempts were seen which used vectors to find the coordinates of the foot of a perpendicular from a vertex to the opposite side. In principle, this is possible but, in this case, the calculations proved too difficult to carry out correctly under examination conditions.

2. The majority of candidates made good attempts at parts (a) to (e) of this question. Many, however, wasted a good deal of time in part (a), proving correctly that $\lambda = \mu = 0$ before obtaining the correct answer. When a question starts "Write down ….", then candidates should realise that no working is needed to obtain the answer. The majority of candidates knew how to use the scalar product to find the cosine of the angle and chose the correct directions for the lines. Parts (c) and (d) were well done. In part (e) , as in Q1(b), the working needed to establish the printed result was often incomplete. In showing that the printed result is correct, it is insufficient to proceed from $\sqrt{416}$ to $4\sqrt{23}$ without stating $416 = 16 \times 26$ or $4^2 \times 26$. Drawing a sketch, which many candidates seem reluctant to do, shows that part (f) can be solved by simple trigonometry, using the results of parts (b) and (e). Many made no attempt at this part and the majority of those who did opted for a method using a zero scalar product. Even correctly carried out, this is very complicated $\left(\mu = \frac{104}{19}\right)$ and it was impressive to see some fully correct solutions. Much valuable time, however, had been wasted.

3. This proved the most demanding question on the paper. Nearly all candidates could make some progress with the first three parts but, although there were many, often lengthy attempts, success with part (d) and (e) was uncommon. Part (a) was quite well answered, most finding $\overrightarrow{AB}$ or $\overrightarrow{BA}$ and writing down $\overrightarrow{OA} + \lambda\overrightarrow{AB}$, or an equivalent. An equation does, however need an equals sign and a subject and many lost the final A mark in this part by omitting the "**r** =" from, say, **r** = 8**i** + 13**j** – 2**k** + λ (2**i** + **j** – 2**k**). In part (b), those who realised that a magnitude or length was required were usually successful. In part (c), nearly all candidates knew how to evaluate a scalar product and obtain an equation in cos θ, and so gain the method marks, but the vectors chosen were not always the right ones and a few candidates gave the obtuse angle. Few made any real progress with parts (d) and (e). As has been stated in previous reports, a clear diagram helps a candidate to appraise the situation and choose a suitable method. In this case, given the earlier parts of the question, vector methods, although possible, are not really appropriate to these parts, which are best solved using elementary trigonometry and Pythagoras' theorem. Those who did attempt vector methods were often very unclear which vectors were perpendicular to each other and, even the minority who were successful, often wasted valuable time which sometimes led to poor attempts at question 8. It was particularly surprising to see quite a large number of solutions attempting to find a vector, $\overrightarrow{CX}$ say, perpendicular to l, which never used the coordinates or the position vector of C.

4. The majority of candidates identified the need for some form of dot product calculation in part (a). Taking the dot product $l_1.l_2$, was common among candidates who did not correctly proceed, while others did not make any attempt at a calculation, being unable to identify the vectors required. A number of candidates attempted to equate l_1 and l_2 at this stage. The majority of candidates, however, were able to show that $q = –3$.

In part (b), the majority of candidates correctly equated the **i**, **j** and **k** components of l_1 and l_2, and although some candidates made algebraic errors in solving the resulting simultaneous equations, most correctly found λ and μ. In almost all such cases the value of p and the point of intersection in part (c) was then correctly determined.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that $\lambda = 1$ at A, $\lambda = 5$ at the point of intersection and so $\lambda = 9$ at B. So substitution of $\lambda = 9$ into l_1 yields the correct position vector –7**i** + 11**j** – 19**k**. A few candidates, by deducing that the intersection point is the midpoint of A and B were able to write down $\frac{9+x}{2} = 1, \frac{3+x}{2} = 7$ and $\frac{13+z}{2} = –3,$ in order to find the position vector of B.

5. This vector question was answered well and many even included the check in part (a) and the statement that a scalar product of zero implied perpendicular lines in part (b). More errors were seen in part (c), as candidates became short of time. However a large number understood the required methods and there were many completely correct responses.

6. Part (a) was answered fairly well and the most common mistake was to have the vectors in the wrong direction, so getting an answer of 4/9. Also vectors *OA*, *OB* and *OC* were sometimes used instead of *AB* and *CB*. In some cases the candidates ignored the question and found the equation of the lines going through *AB* and *BC* instead. A minority of candidates did this part using the cosine rule and the lengths of the three sides of the triangle.

Part (b) was not popular and was frequently omitted, but a significant number of the candidates tried to apply the ½*a* sin *C* formula. In most cases the two lengths were correct, but relatively few found an exact value for the sine of the angle. Hero's formula was rarely seen. Some recognised that the triangle was isosceles and then attempted to find the height by Pythagoras theorem or trigonometry. If these latter methods were used, often the exact answer was not found.

In part (c) most candidates tried to use the scalar product to establish that the lines were perpendicular, sometimes altering correct vectors in an attempt to fix the answer.

There was some difficulty in part (d) with the concept of the ratio of lengths of vectors. Even those with two correct vectors did not necessarily produce ratio form of the answer. Many found the lengths of the two vectors and used this to find the ratio, which was acceptable.

7. No Report available for this question.